CORE MATHEMATICS (C) UNIT 2 TEST PAPER 8

1. The area of the cross-section of a tunnel of width 3.5 m was estimated using the following measurements, all taken from one side wall:

Distance from side (m)	0	0.5	1	1.5	2	2.5	3	3.5
Height (m)	1.1	1.6	1.7	1.9	1.9	1.7	1.6	1.1

Use the trapezium rule with the given data to estimate the area of the cross-section.. [5]

2. In the triangle ABC, $AB = 3$ cm, $BC = 5$ cm and $CA = 6$ cm. Calculate
   (i) the smallest angle of the triangle, in radians to 2 decimal places, [3]
   (ii) the area of the triangle, in cm^2 to 1 decimal place. [2]

3. (i) Expand $(3 - x)^5$ by the binomial theorem, simplifying each term. [5]
   (ii) Hence find the expansion of $(3 - x - x^2)^5$ in ascending powers of x as far as the term in x^2. [3]

4. Given that $(x - 3)$ is a factor of $f(x) \equiv 2x^3 - 3x^2 + kx + 6$,
   (i) find the value of the integer k. [3]
   (ii) Factorise $f(x)$ completely. [3]
   (iii) Find the remainder when $f(x)$ is divided by $(2x - 3)$. [2]

5. Solve, for $0 \leq x \leq 360$, the equations
   (i) $\tan (2x - 45)^\circ = 1$, [4]
   (ii) $2 \sin^2 x^\circ + \cos x^\circ = 1$. [5]

6. The gradient G of a curve at the point (x, y) is given by the formula
$$G = 6x^2 + 2x - 3.$$
The curve passes through the origin.
   (i) Find the coordinates of the other two points where the curve crosses the x-axis. [7]
   (ii) Sketch the curve. [3]

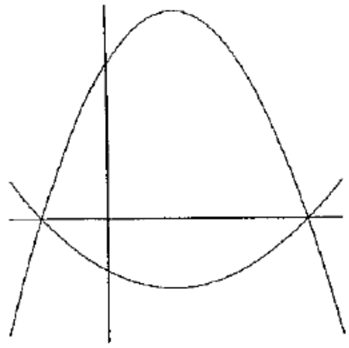
7. The diagram shows the curves with equations

$y = x^2 - 4x - 12$ and $y = k(x^2 - 4x - 12)$, where

k is a negative integer.

The distance between the maximum point on one curve

and the minimum point on the other curve is 64 units.



(i) Find the value of k. [5]

(ii) Find the area of the finite region contained between the two curves. [7]

8. (i) The first three terms of an arithmetic sequence are $-4, x, 5$. Find

(a) the value of x, [2]

(b) the 11^{th} term of the sequence, [2]

(c) the smallest value of n for which the nth term of the sequence is more than 100. [5]

(ii) The sum of the first two terms of a geometric series is 36 and the sum to infinity is 38.4.
Find the two possible values of the common ratio and of the first term of the series. [6]

CORE MATHS 2 (C) TEST PAPER 8 : ANSWERS AND MARK SCHEME

1.	Area $\approx \frac{1}{2}(\frac{1}{2})(2.2 + 2(10.4)) = 5.75\ \text{m}^2$	M1 A1 M1 A1 A1	5
2.	(i) $\cos C = (25 + 36 - 9)/60 = 13/15$ Angle $ACB = 0.52$ radians	M1 A1 A1	
	(ii) Area $= 15 \sin 0.52 = 7.5\ \text{cm}^2$	M1 A1	5
3.	(i) $(3-x)^5 = 3^5 + 5(3^4)(-x) + 10(3^3)(-x)^2 + 10(3^2)(-x)^3 + 5(3)(-x)^4 - x^5$	M1 A1 A1	
	$= 243 - 405x + 270x^2 - 90x^3 + 15x^4 - x^5$	M1 A1	
	(ii) $(3 - x - x^2)^5 = 243 - 405(x + x^2) + 270(x + x^2)^2 - \ldots = 243 - 405x - 135x^2$	M1 A1 A1	8
4.	(i) $54 - 27 + 3k + 6 = 0$ $k = -11$	M1 A1 A1	
	(ii) $f(x) = (x-3)(2x^2 + 3x - 2) = (x-3)(2x-1)(x+2)$	M1 A1 A1	
	(iii) Remainder $= f(3/2) = 27/4 - 27/4 - 33/2 + 6 = -21/2$	M1 A1	8
5.	(i) $2x - 45 = 45, 225, 405, 585$ $x = 45, 135, 225, 315$	M1 A1 M1 A1	
	(ii) $2 - 2\cos^2 x + \cos x = 1$ $2\cos^2 x - \cos x - 1 = 0$	M1 A1	
	$(2\cos x + 1)(\cos x - 1) = 0$ $x = 0, 120, 240, 360$	M1 A1 A1	9
6.	(i) Integrating, $y = 2x^3 + x^2 - 3x + c$ $y(0) = 0$ so $c = 0$	M1 A1 A1	
	When $y = 0$, $x(2x+3)(x-1) = 0$ Points are $(-3/2, 0)$, $(1, 0)$	M1 M1 A1 A1	
	(ii) Curve sketched, crossing axes at $(-3/2, 0)$, $(0, 0)$, $(1, 0)$	B3	10
7.	(i) Turning points where $2x - 4 = 0$ $x = 2$ $y = -16, y = -16k$	M1 A1 A1	
	Difference $= -16(k-1) = 64$, so $k = -3$	M1 A1	
	(ii) Curves cut x-axis where $(x+2)(x-6) = 0$ $x = -2, x = 6$	M1 A1	
	$-4\int_{-2}^{6}(x^2 - 4x - 12)\,dx = -4\left[\frac{x^3}{3} - 2x^2 - 12x\right]_{-2}^{6} = -4\left(-72 - \frac{40}{3}\right) = \frac{1024}{3}$	M1 A1 A1 M1 A1	12
8.	(i) (a) $x = (5-4)/2 = \frac{1}{2}$ (b) $T_{11} = -4 + 10(4.5) = 41$	M1 A1 M1 A1	
	(c) $T_n = -4 + 9/2\,(n-1) > 100$ when $n - 1 > 208/9$ $n = 25$	M1 A1 M1 A1 A1	
	(ii) $a + ar = 36$, $a/(1-r) = 38.4$	B1 B1	
	$38.4(1-r)(1+r) = 36$ $1 - r^2 = 15/16$ $r = \pm 1/4$	M1 A1	
	$a = 36/(5/4) = 28.8$ or $a = 36/(3/4) = 48$	M1 A1	15